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INTERACTION OF LARGE AMPLITUDE STRESS
WAVES IN LAYERED ELASTIC-PLASTIC MATERIALS

FINAL REPORT

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Nonlinear wave interactions; shocks; impact loading; elastic-
plastic materials. Fracture; nonlinear elastic deformations.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Mathematical techniques are developed to analyze the nonlinear
wave interactions that occur when a uni-directional wave traveling
through an hysteretic material arrives at an interface with some
other hysteretic material. Also, mathematical techniques are
developed to describe the deformations of finitely strained mate-
rials surrounding cracks, notches, and inclusions.

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1. FORWARD

In a series of papers [1], [2], [3], [4] E. Varley and his co-workers developed mathematical techniques that can be used to analyze the diverse nonlinear wave interactions that can occur when a slab of elastic-plastic material is finitely deformed by plane waves propagating in directions normal to the parallel interfaces bounding the slab. Typically, the slab could be contained between two other different elastic/plastic materials that are of semi-infinite extent in the direction of wave propagation, or the slab could be just one layer in a laminated (multi-layered) material. The waves can either be pure stretching waves or, if the materials are incompressible, linearly polarized shear waves.

This work was in two parts. In part I, which is described in [1], the reflection and transmission of a large amplitude pulse when it arrives at an interface with some other material was analyzed. First, the ideas of nonlinear impedance, reflection coefficient and transmission coefficient for such an interface were introduced. Then, these were used to determine the amplitudes of the reflected and transmitted pulses in terms of that of the incident pulse. The algorithms obtained were quite general: no restriction was placed on the stress-strain relations of the materials separated by the interface. The results for a single interface were then used to analyze the decay in the amplitude of a pulse as it moves back and forth in a slab contained between two interfaces separating the slab from materials that are of semi-infinite extent. The disturbance in the slab is caused by the arrival of a stress pulse traveling through one of the adjacent materials. The decay occurs because at each contact with the interface part of the energy of the trapped pulse is radiated to the adjacent materials. The laws obtained have simple graphical interpretations.

In part II of their study, which is described in [2], [3] and [4],
Varley and his co-workers gave a more detailed account of the deformation
produced in an elastic/plastic slab when one boundary is impulsively loaded
while the other radiates energy to an adjacent material. This was correlated,
both qualitatively and quantitatively, by a family of stress/strain laws for
which the governing nonlinear equations can be solved exactly. The materials
include polycrystalline solids at pressures up to the yield stress, metals
when subjected to pressures in the hydrodynamic range, water, explosive products,
gases, as well as elastic/plastic, rigid/plastic and rigid/elastic materials.
The results reported in [2] were obtained by showing that for these model
materials a simple, but exact, representation can be found that describes the
interaction of a centered wave with any wave traveling in the opposite
direction. The arbitrary functions occurring in this representation were
then determined for the special case when the opposite traveling wave is the
wave reflected from an interface with some other elastic material during the
arrival of the centered wave. It was shown that this same problem arises
during the early stages of a wide variety of technically important deformations
that are produced by sudden loading, even when shocks are formed initially.
One example is the initial stage of the deformation produced in a slab when
a constant strength shock wave traveling through an adjacent material arrives
at their common interface. Another is when a 'hitter' bar, or plate, which is
composed of two materials separated by a distinct interface, impacts some other
material. In general, only at the subsequent reflection of the disturbance
as a continuous wave from the loaded boundary do the characters of these
deformations begin to differ. Usually, it is at this second reflection that
the peak stresses and strains occur. In [3] it was shown how to analyze this
second reflection in two limiting cases: when the first reflection is from a

perfectly free boundary. This was achieved by obtaining an exact representation for the interaction of these reflected waves with any opposite traveling wave generated at the loaded boundary. This general representation was used in [3] to obtain a detailed description of the early stages of the deformation in the slab when the load generating the disturbance varies after changing discontinuously.

The general aim of the proposed program of research was to continue the development of mathematical techniques that can be used to analyze the interactions of large amplitude waves in elastic/plastic materials. Special attention was to be given to those interactions that result when strong waves propagate through layered media; those that result from the presence of strong shocks; those that result from the presence of both elastic and plastic zones and those that result from combined stress. Also, it was intended to make a preliminary investigation of the nonlinear wave interactions that can occur during the dynamic fracture of an elastic material.

2. SUMMARY OF COMPLETED RESEARCH

2.1 Nonlinear wave interactions

Techniques developed by E. Varley and his co-workers (see, for example, [1]-[4]) have been used to analyze the diverse nonlinear wave interactions that can occur when a slab, or bar, of elastic-plastic material is finitely deformed by plane stretching waves. A typical example of such a deformation is depicted in figure 1. A plane stress wave traveling from right to left through an elastic-plastic (hysteretic) material arrives at a plane interface separating this material from some other elastic-plastic material where it is partially reflected and partially transmitted. The direction of wave propagation is normal to this interface. For a slab this direction is the only principal direction in which the principal strain e varies (uni-axial strain), for a rod or a bar the direction of wave propagation is the only direction in which the principal stress σ varies (uni-axial stress). For both situations σ, e and u , the component of material velocity in the direction of wave propagation, are functions only of time t and the Lagrangian distance X measured from the material interface X=0 ; they are related by the equations

$$\frac{\partial \sigma}{\partial x} = \rho_0 \frac{\partial u}{\partial t} \quad \text{and} \quad \frac{\partial u}{\partial x} = \frac{\partial e}{\partial t} \tag{1}$$

where ρ_0 , the density of the material before the arrival of the wave is constant, but different, on either side of the interface.

For definiteness, consider the situation when the incident pulse is compressive and both materials are hysteretic but rate independent. Figues 2a depicts a typical relation between σ and e at any particle when the deformation is one of plane strain and figure 2b depicts the relation between σ and e for the case of plane stress. Over portions of the curves where arrows point in both directions the material behaves elastically, where

arrows point in one direction the material is non-elastic. During loading σ was determined from e by a relation of the form

$$\sigma = L(e) \tag{2}$$

while during unloading σ and e are related by an equation of state of the form

$$\frac{\partial \sigma}{\partial t} = A(\sigma)\frac{\partial e}{\partial t} . \tag{3}$$

In (3) the function $A(\sigma)$ was assumed to satisfy a relation of the form

$$\frac{dA}{d\sigma} = \mu A^{-\frac{1}{2}} + \nu A^{\frac{1}{2}} \tag{4}$$

where (μ, ν) are material constants. Then, following Varley and his co-workers, equations (1) and (2) were integrated in closed form and the representations obtained were used to analyze some simple, but technologically important, deformations which occur during impact and which involve unloading.

One deformation that has been considered in some detail is that which occurs when the incident pulse is in the form of a shock followed by a rarefraction wave. Then, when the materials are in the form of slabs the transmitted wave is a simple wave in which (2) holds followed by an unloading wave in which (3) holds. To date this problem has only been analyzed analytically when $A \equiv$ constant in the unloading case. However, as shown by Bell and Moon [5], in practice $A(\sigma)$ varies appreciably during unloading. The experimental A versus σ relation obtained by Bell and Moon [5] has been fitted by relations satisfying (4) and the effect of the nonlinearity of the unloading curve has been analyzed. A paper describing this aspect of the research is being prepared.

2.2 Finite static deformations of elastic materials enclosing cracks, notches and inclusions

As a preliminary part of the investigation of the nonlinear wave interactions that occur during the dynamic fracture of an elastic material we have analyzed a wide variety of static deformations of finitely strained elastic materials which surround cracks, notches and inclusions.

The main success in constructing exact, non-trivial, solutions to the equations governing the equilibrium states of finitely deformed elastic materials has been obtained in the incompressible limit when the geometric symmetries of the deformation, coupled with the constraint of incompressibility, allow one to determine the strain field independently of the precise form of the relation between stress and strain. This fact was exploited by Rivlin and his co-workers (see, for example, (8) and (9)) to solve a wide variety of kinematically determinate problems without specializing the form of the strain energy function of the material. However, as soon as one considers problems that are kinematically indeterminate there are few exact solutions, even for special forms of the strain energy function. A notable exception was found by Fritz John (10) who pointed out that for a very special class of compressible materials, which he termed harmonic materials, the governing equations simplified considerably. Although this simplification allowed one to construct exact solutions to the governing equations, the problem of determining solutions which also satisfy prescribed auxiliary conditions has received little attention. In the course of research sponsored by this contract, we have generalized Fritz John's earlier work and have shown that for special forms of the stress versus strain relation, which correspond to physically reasonable behavior, the governing equations can be integrated and several problems of technical interest can be solved exactly. In particular,

the plane strain, or plane stress, deformation which is produced when an
infinite material containing a stress-free elliptical hole is subjected to
a pure homogeneous deformation at infinite has been analyzed in detail.
The integration techniques has also been extended to holes of other shapes.
The special case when the ellipse degenerates to a slit has received special
consideration because then, our analyses are directly applicable to the
study of fracture or tearing. The interplay between nonlinearity and geometric
effects due to the fact that, in practice, the tip of the crack has a non-zero
radius of curvature has been discussed in detail. Two papers on this aspect
of our work have been accepted for publication.

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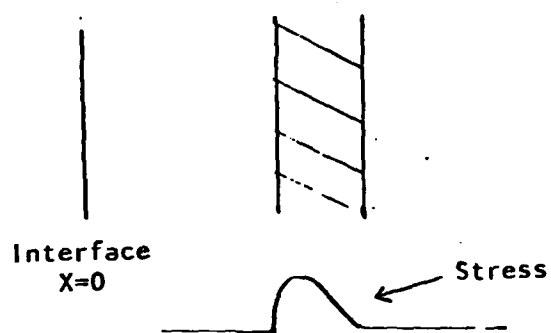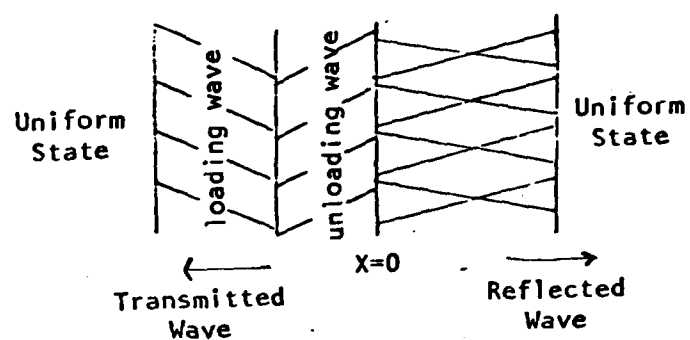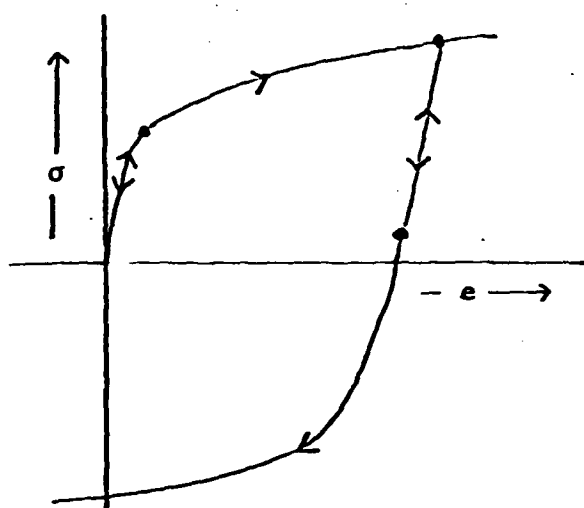
FIGURES

Figure 1a

Figure 1b

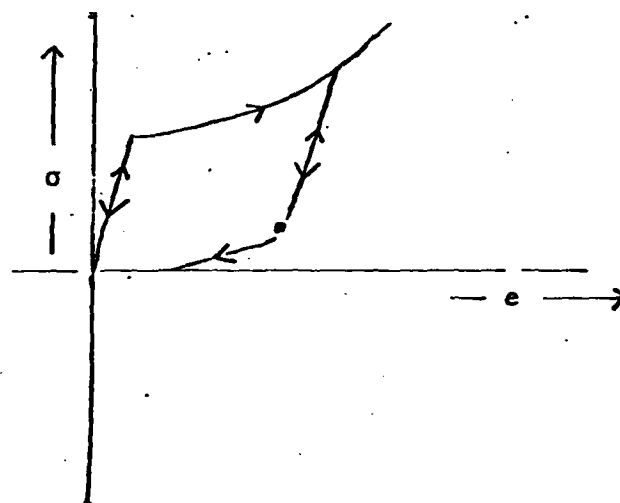Figure 2a. Typical nominal stress σ versus strain e in uniaxial strain.

Figure 2b. Typical nominal stress σ versus strain e in uniaxial stress.

APPENDIXES

Papers accepted for publication:

Varley, E. and Cumberbatch, E. 1977 Finite Elasticity A.S.M.E. publication
  vol. 27.

Varley, E. and Cumberbatch, E. 1980 "Finite deformations of elastic materials
  surrounding cylindrical holes" publication pending in Journal of Elasticity.

Papers in preparation:

Varley, E. "Nonlinear wave interactions in hysteretic materials".

Books:

"Propagation of Shock Waves in Solids" Ed. by E. Varley A.S.M.E. publication
  AMD-Vol. 17.

Invited lectures:

"Nonlinear wave interactions in hysteretic materials" A.S.M.E. Applied
  Mechanics Conference, Salt Lake City, June 1976.

"The finite deformation of an elastic material surrounding an elliptical hole"
  A.S.M.E. Winter Meeting, Atlanta, 1977.